Derivation of the veloclinic Mean Maximal Power Duration Models

Background

The power-duration curve of athletic performance is determined by the maximal work that can be performed over a given length of time. The modern models of the power duration relationship stem predominantly from the Critical Power (CP) Model (Monod and Scherrer 1965, Moritani 1981, Hill 1993). This model states that the curve is determined by the sum of two contributing components; CP which is a power that can be sustained without fatigue for a very long period of time, and W' which is a finite work capacity that is instantaneously available:

$$P(t) = W'/t + CP$$

This mathematical model produces a hyperbolic curve which has been mapped to underlying physiological processes in a broad range of studies (Jones, 2010). The model however fails outside a relatively narrow time range. Very short durations produce predictions of infinitely high rates, and long durations produce predictions of infinitely sustainable rates (Morton, 2006).

Attempts to extend the model have included short duration rate limits imposed by linear and exponential feedback mechanisms:

$$P(t) = W' * [t - (W'/(CP - Pmax))] + CP$$ (Morton, 1996)

$$P(t) = Pmax * e^{(-t/tau)} + CP * (1 - e^{(-t/tau)})$$ (Ward-Smith, 1985)

These modifications improve the predictive power at short duration but do not address the CP model failure and long durations.

Logarithmic and exponential rate decays have thus been applied to the CP components in attempts to further extend the model for long durations.

$$P(T) = [S/T * (1 - e^{(-T/20)})] + 1/T \int_0^T [BMR + B * (1 - e^{(-t/30)})]dt, \; (T < 420s)$$

or

$$P(T) = [A + [Afln(T/420)]/T * (1 - e^{(-T/20)})] + 1/T \int_0^T [BMR + [B + E * ln(T/420)] * (1 - e^{(-t/30)})]dt$$

$, (T > 420s)$ (Peronnet and Thibault, 1989)

$$P(t) = Pmax * [tau1/t * (1 - e^{(-t/tau1)}) - tau1a/t * (1 - e^{(-t/tau1a)}) + CP * [tau2n/t * (1 - e^{(-t/tau2n)}) - tau2a/t * (1 - e^{(-t/tau2a)})], \; (solved\ with\ 3\ relaxations\ of\ tau2n)$$

(Alvarez, 2002)

These extended models allow better fits across broader ranges but at the expense of significant complexity and over-reliance on apriori fixed parameters for solvability which may not generalize well for some individuals.

In practice, the original critical power model is still often favored over the extended range models. It appears that extended range does not outweigh the appeal of a simple intuitive model.

To address these pragmatic barriers new candidate models are proposed here that employ the intuitive CP concept of finite capacities with simple forms of linear and exponential feedback. These novel two component models offer extended range, minimized complexity, and intuitive parameters.

The new models are derived as follows:

First, let $P(t)$ represent the average power at any duration t as the sum of two contributing components. Let W' represent a finite capacity in joules where $W'1$ represents a low capacity fast twitch component and $W'2$ represent a high capacity slow twitch component.

$$P(t) = W'1/t + W'2/t$$

Compared to the CP model, W' is conserved as $W'1$. However, CP as an infinitely sustainable rate is replaced by a second component with a limited capacity. The repetitive structure of the model is in keeping with the goals of simplicity and intuitiveness as the capacity limited structure of the first component is carried over to the second component as well.

Next, a simple feedback can be applied to both components to reflect a rate limitation for each system. Starting with a simple linear feedback, let tau represent a time constant that describes the rate limitation of each component as a function of time. The final equation now becomes:

$$P(t) = W'1/(t+tau1) + W'2/(t+tau2)$$

This equation is the simplest mathematical description of the extended power-duration curve currently proposed.

The equation can also be rewritten in terms of maximal rates. Let Pow represent the maximal theoretical rate of each component so that the capacity of the component equals the maximal rate multiplied by the time constant:

$$W' = Pow * tau$$

Substitution yields:

$$P(t) = Pow1 * tau1/(t+tau1) + Pow2 * tau2/(t+tau2)$$

Simplification yields:

$$P(t) = Pow1/(1+t/tau1) + Pow2/(1+t/tau2)$$

The power curve is now fully described by two components, each with a limited capacity and maximal rate related by a time constant tau. The model thus describes the complete curve as a continuum of sequential regions that can be quantified by the fast twitch rate, fast twitch capacity, slow twitch rate, and slow twitch capacity parameters respectively.

An alternate form of the model can also be derived which allows for a delay in the depletion of the slow twitch capacity. Let $beta$ describe the delay in capacity depletion as a power function of time:

$$P(t) = W'1/(t+tau1) + W'2/(tau2 * (1+t/tau2)^{1/beta})$$

Substitution yields:

$$P(t) = Pow1/(1+t/tau1) + Pow2/(1+t/tau2)^{1/beta}$$

With minimal additional complexity, this alternate model allows for the intake of food during extended efforts which delays the depletion of the slow twitch capacity. Further, if $tau2$ is constrained to 3600 - 7200 seconds, the model predicts a primary slow twitch limiter with a delayed depletion at decreasing total rates analogous to the metabolism of carbohydrate and fat respectively.

Similarly, the two component concept above can be described using simple exponential feedback as well. Again, let tau represent a time constant that describes the rate limitation of each component as a function of time. The final equation now becomes:

$$P(t) = W'1/t * (1-exp(-t/tau1)) + W'2/t * (1-exp(-t/tau2))$$

The equation can also be rewritten in terms of maximal rates. Let Pow represent the maximal theoretical rate of each component so that the capacity of the component equals the maximal rate multiplied by the time constant:

$$W' = Pow * tau$$

Substitution yields:

$$P(t) = Pow1 * tau1/t * (1 - exp(-t/tau1)) + Pow2 * tau2/t * (1 - exp(-t/tau2))$$

The power curve is now fully described by two components, each with a limited capacity and maximal rate related by a time constant tau. Note this model is mathematically similar to the Alvarez model with but with simplification allowed by modelling components by twitchedness rather than oxygen dependence.

Lastly, it is possible interchange components so that additional forms are easily derived from testing.

<u>Works Cited</u>

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